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Dependence Structure and Portfolio Value-At-Risk for Some Selected Nigerian Stocks Using a Copula-Based Volatility Model

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Abstract
The study examined dependence structure and estimates portfolio risk on data from some selected Nigerian stocks. Marginal model for the stock returns and a joint model for the dependence for the dependence were specified. EVT model was employed for the marginal distribution of each return series, and for the joint model, the family of copula such as Gaussian, Frank, Gumbel, Clayton, BB7, Student-t copula were used with difference dependence structure, Using LL, AIC, and BIC values, BB7 is found to be the best fitted copula. Copula was used to measure Portfolio risk and global minimum risk portfolio is selected based on efficient frontiers. In estimating VaR, precise specification and identification of the probability of an extreme movement in the value of an individual asset (or portfolio) is essential for risk assessment. The evidence has direct implications for investors and risk managers during extreme currency market movements.

***Key words**: Dependence structure, portfolio value-at-risk, copula, volatility, extreme value*

1.1: INTRODUCTION

Previous studies have shown that measuring linear correlation can lead to misleading interpretations and dependency effects when it comes to financial variables. Existing studies have shown that the copula approach can be adopted as an alternative and effective way of analysing the dependence structure between financial assets because it provides the solution to the fat-tailed problems in multivariate cases arising from the possibility of large co-dependencies or extreme movements. Focus on the application of copula-based forecasting models, studying tail dependence and how the choice of a risk model affects asset allocation. Analysis of the dependence structure of weekly returns of some selected Nigerian financial assets using a modelling approach.

Data on financial returns have been shown to be non-normally distributed, compared to the classical assumption of a normal distribution. Extreme value theory (EVT) is adopted to show extreme risks through tail modelling, which is important for risk measurement. To accurately describe the high dependency relationship between assets, the extreme value theorem is introduced to construct the marginal distribution. The effects of leverage and long tails have been clearly observed in some selected Nigerian financial statements. Assets in the portfolio include Dangote Cement, Nestle, Vitaform, Fidson and Guarantee Trust Holdings Company. Using some selected Nigerian stock markets, an analysis of the performance of various risk model portfolio strategies indicates that EVT forecast models, which use Gaussian ot Student COPULAS, are better at minimizing portfolio risk. Use COPULA functions to model the dependence of large market movements and test the validity of the results by implementing back testing techniques. The results show that the COPULA-based approach does not vary significantly across tests using information criteria, provides better estimates than common methods currently used, and captures VaR well based on differences in the number of exceptions generated during different testing periods. observation with the same confidence level. The results show that the COPULA-based approach does not vary significantly across tests using information criteria, provides better estimates than common methods currently used, and captures VaR well based on differences in the number of exceptions generated during different testing periods. observation with the same confidence level. The VaR model measures market risk by determining how much the value of a portfolio will decline over a given period with a given probability because of changes in market rates or prices. VaR is the standard for measuring market risk and has become a widely used tool in risk management by financial institutions and regulators over the last two decades (Nieto & Ruiz, 2016). There are many methods for estimating VaR (e.g. Holton (2003), Jurion (2007), Malls (2011)) and the most widely used method is the covariance method (developed by JP Morgan using its own risk metrics in 1993), historical simulation and Monte Carlo simulation. The variance-variance and historical simulation methods assume that asset returns are independently and normally distributed. This assumption contradicts the empirical evidence presented by Sheikh and Qiao (2010), which shows that in many cases, financial asset returns are neither independent nor normally distributed, but are in fact volatile and highly volatile, which leads to an underestimation or overestimation of VaR; This is because extremely large positive and negative asset returns are more common in practice than in normally distributed models. Extreme value theory (EVT) provides a set of methodological tools to address issues such as skewness, large tails and rare (extreme) events, and is used to calculate measures related to tails, which are about showing extreme risks. using backward models, which is important for risk management. It is necessary to introduce the extreme value theorem to construct the marginal distribution to accurately describe the extreme dependence relationship between assets. Modeling the dependence between returns has played an important role in portfolio optimization, credit risk and financial market analysis. One difficulty in estimating the value at risk (VaR) of a portfolio is modeling the comovement of returns, that is, the dependence structure. VaR has to do with the tail of the distribution. A copula is an indirect measure of risk that captures the dependency between extreme events, and the advantage of a copula is that it is used as a measure of the dependency structure and tail dependency. Initially, the dependence structure between random variables is completely described by a joint distribution function (linear correlation, as a criterion, captures part of this dependence structure. It has the disadvantage that linear correlation is not constant under nonlinear monotonic transformations of random variables. Linear correlation conflicts with an important property of copulas, that they are constant under monotonic transformations of random variables). The multivariate dependence structure between markets can be modeled using copulas that adapt well to nonlinear dependencies and nonnormal distributions. Implicit Gaussian copulas are used in the literature, but it is important to consider copulas other than implicit Gaussian copulas due to the failure of the correlation approach to capture the dependence between extreme events, as shown by Longin and Solnick (2001), Bai et al. (2003) and Hartmann *et al.* (2004). Copulas provide financial risk managers, investors, and regulators with a powerful tool to model the dependency between different elements of a portfolio, which is better than the traditional correlation-based approach.

2.1 Literature Review

A multivariate model consists of two components: a univariate or marginal model that describes each variable, and a dependence structure among these marginal variables. Since the 1960s, Mandelbrot (1963) and Fama (1965) have shown that univariate asset return distributions are not normally distributed, as evidenced by excessive kurtosis (or "fat tail") and skewness above the normal distribution. I've been paying attention to what isn't there. Numerous empirical studies have investigated the interdependence between stock markets. For example, Longin and Solnik (2001) and Ang and Chen (2002) use linear correlation to show that the correlation between stock market returns in both international and domestic markets is not constant over time.

Most of the early studies used linear correlation coefficients as a measure of reliance on stock markets and financial econometrics in general. It is now well known that linear correlation is a natural measure of dependence only for spherical and elliptical distributions, including multivariate normal distributions, but real-world distributions rarely belong to this class. None (Embrechts, McNeil, & Straumann, 2002). Linear correlation is invariant under strictly increasing linear transformations, which is a desirable property. However, linear correlation is not a measure of agreement because it is not invariant under nonlinear, strictly incremental transformations (Embrechts, Lindskog, & McNeil, 2003).

Jondeau and Rockinger (2003) investigate the behavior of some stock market returns based on highly sloped distributions and show that linear correlation is not an ideal measure of dependence. Therefore, assuming multivariate normality or using linear correlation coefficients to measure stock market dependence can lead to inaccurate hedge selection, incorrect portfolio decisions, or underestimation of portfolio risk. may lead to. In addition to being non-normally distributed, stock returns depend more on the extremely negative low tail than on the extremely positive high tail. This is a phenomenon that cannot be captured by linear correlation (Longin and Solnik, 2001; Ang and Chen, 2002). Poon *et al.* (2004) reports an asymmetric dependence of asset returns, which represents a rejection of multivariate normality. As Patton (2006) showed using the copula function, with asymmetric dependence, returns are more correlated during market crashes than during market booms. Kendall's Tau and Spearman's Rho as copula measurements are the best substitutes for linear correlation coefficients (Embrechts *et al.*, 2003). A desirable feature of Kendall's Tau and Spearman's Rho is that they are invariant to strictly increasing component-wise transformations. Another desirable feature is that for continuous random variables, all values in the interval [O;1] of Kendall's Tau or Spearman's Rho can be obtained by appropriate selection of the underlying copula; This is not the case for linear correlations. McNeil, Frey, and Embrechts, (2005), and Embrechts *et al.* (2002) propose a copula-rank correlation to model the concept of dependency. McNeil *et al.* (2005) explain why copula has proven to be very versatile in risk management due to several reasons. First, extreme dependencies between assets can be explained by copulas, regardless of whether the dependencies are assumed to be constant or change over time. Second, the copula approach allows the connection or "coupling" of multiple boundary models with different possible dependency specifications. Finally, tail dependencies, both symmetric and asymmetric dependencies, and positive and negative dependencies, can be easily captured by copulas. As a result, Copula has become a very popular method for modeling dependencies between assets and markets. A copula is a function that connects a multivariate distribution to its one-dimensional tail (Sklar, 1959). According to Nelsen (2006), the copula can be viewed from two perspectives. "From one perspective, a copula is a function that connects or combines a multivariate distribution function with her one-dimensional marginal distribution function." Alternatively, a copula is a multivariate distribution function whose one-dimensional tail is uniform in the interval (0,1).

Modelling dependencies based on available information leads to the study of conditional copulas. To model an unconditional or time-varying copula, you must specify a model for the unconditional or conditional marginal distribution of the standardized residuals. As pointed out by Patton (2006), directly modeling the dependence structure of variables using unconditional probabilities yields a model of the unconditional copula of returns. Therefore, in this article, we use Gaussian, Frank, Gumbel, Clayton, BB7, Student t copulas with differential dependence structure and use LL, AIC, and BIC to determine the optimal model.

3.1 Methodology

3.1.1 Extreme Value Theory (EVT)

Let y_i be the standardised residuals that are extracted from the GARCH models. The peak over threshold (POT) data selection approach used to fit the Generalised Pareto Distribution (GPD), and the block maxima (BM) data selection approach for fitting the Generalised Extreme Value Distribution (GEVD) are used to model the standardised residuals from the selected GARCH models.

3.1.2 The peak over threshold (POT)

This method has become the method of choice in financial applications. defining a sequence of values that exceed a high threshold u, according to the POT approach. The distribution of excess values is given by:

$$F_u(y) = \frac{\Pr(X-u \leq y, X>u)}{\Pr(X>u)}, \quad 0 \leq y \leq x_F - u$$

where $y = y - u$ is the excess over u and x_F the right endpoint of F

According to Pickands (1975) the limiting distribution of F_u can be approximated by a **3.1.3 Generalized Parato Distribution (GPD)** given by

$$G_{\xi\varphi}(y) = \begin{cases} 1 - \left(1 + \frac{\xi(y-u)}{\varphi}\right)^{\frac{-1}{\xi}}, & \text{if } \xi \neq 0 \\ 1 - e^{\frac{-(y-u)}{\varphi}}, & \text{if } \xi = 0 \end{cases} \qquad (1)$$

where $y > u, y - u$ is the exceedance , and $0 \leq y \leq -\frac{\varphi}{\xi}, \varphi > 0,$

$G_{\xi\varphi}(y)$ *is the GPD with* ξ the shape parameter, and φ the scale parameter and a threshold u. The value of ξ shows how heavy the tail is, with a biggest positive value (ξ) indicating a heavy tail. $\xi < 0$ indicating a bounded tail, $\xi = 0$ indicates a light tail.

The density function is given as:

$$G_{\xi\varphi}(y) = \begin{cases} \frac{1}{\varphi}\left(1 + \frac{\xi(y-u}{\varphi}\right)^{-1} & \text{, if } \xi \neq 0 \\ exp^{\frac{-(y-u)}{\varphi}}, & \text{if } \xi = 0 \end{cases} \quad (2)$$

3.1.4. The Generalised Extreme Value Distribution (GEVD)

The GEVD is the limiting distribution of normalised block maxima of a sequence of independent identically distributed random variables. The GEVD is given as follows:

$$G_{\xi,\mu,\varphi}(y) = \begin{cases} \exp\{-\left(1 + \xi\left(\frac{y-u}{\varphi}\right)\right)\} & \text{, if } \xi \neq 0 \\ \exp\{-\exp(-(\frac{y-u}{\varphi}), & \text{if } \xi \to 0 \end{cases} \quad (3)$$

with $\xi \neq 0, \varphi > o$ and $1 + \xi\left(\frac{y-u}{\varphi}\right) > 1$

The distribution F can be expressed as function of the conditional excess distribution over the threshold u as follows:

$$F(x) = [1 - F(u)]F_u(y) + F(u).$$

The function of $F(u)$ can be estimated non-parametrically by $\frac{n-k}{n}$, where n is the total number of observations, and k is the number of observations above the threshold u, using the method of historical simulation (HS). After replacing $F_u(y)$ by $G_{\xi\psi}(y)$,we get the following estimate for $F(x)$: $F(x) = 1 - \frac{k}{n}[1 + \xi\frac{x-u}{\psi}]^{-\frac{1}{\xi}}$

For $X > u$,where **ξ** and**ψ** can be estimated by the method of maximum likelihood. The EVT approach described above focuses directly on the tail but does not acknowledge the fact that financial asset returns are non-iid. Most financial return series exhibit volatility clustering, fat-tailed distributions, and leverage effect. While the fat tails might be modeled directly with EVT, the lack of iid returns is problematic. One approach to this problem is provided by McNeil and Frey (2000). Using a two-stage approach they estimate the conditional volatility using a GARCH model in stage one. The GARCH model serves to filter the return series such that GARCH residuals are closer to iid than the raw return series. Even so, GARCH residuals have been shown to exhibit fat tails. In stage two, EVT is applied to the GARCH residuals. As such, the GARCH-EVT combination accommodates both time-varying volatility and fat-tailed return distributions.

3.2 Portfolio risk problem

Let us consider the problem of measuring the risk of holding a portfolio consists of N assets with returns at T-th day, denoted as $x_{n,T}$, given the historical data

$$\{x_{n,t} | t = 1,2, \ldots., T-1\}, \text{ for } n = 1,2, \ldots.. N.$$

The portfolio return at t-th day, denoted as x_t, is approximately equal to

$$x_t = \omega_1 x_{1,t} + \omega_2 x_{2,t} + \cdots + \omega_N x_{N,t}$$

Where ω_n is the portfolio weighs of asset n and

$\sum_{n=1}^{N} \omega_{n,t} = 1$, for $t = 1,2, \ldots . . T$,

$n = 1,2, \ldots , N$.

Morgan(1994) published a risk control method called Riskmetrics, which is mainly based on a parameter named Value at Risk(VaR). For a given time horizon $T$and confidence level p, the VaR is defined as the loss in market value over the time horizon T that is exceeded with probability $1 - p$. Precisely, VaR of a portfolio can be defined as follows.

Definition. Let $H_T(x_T|\mathfrak{J})$ be the conditional distribution function of the returns of portfolio consists of $x_1, x_2, \ldots , x_N$ at time T with conditional set $\mathfrak{J}$.

$$\mathfrak{J} = \left\{ \left\{ \{x_{n,t}|n = 1,2, \ldots . , N\}, t = 1,2, \ldots . , T - 1 \right\} \right\}$$

$\mathfrak{J}$ represents the past information from day 1 to day $T - 1$.

Then the VaR of the portfolio at time T, with confidence level p, where $p \in (0,1)$ is defined by

$$VaR_{T(p)} = inf\{s =: H_T(s|\mathfrak{J}) \geq 1 - p\}.$$

3.3 Copula

A copula is a probability model that represent a multivariate uniform distribution which combines the marginal distributions to the joint distribution function F. This can be done by specifying the marginal distribution function F and the copula function C . Sklar (1959) introduced the concept of copulas that has been recognized as a powerful tool for modelling dependence between variables. Applications based on copula theory centralize around the Sklar theorem which ensures the relation between a N-dimensional distribution and a corresponding copula. A copula function in N dimensional whose domain is $[0,1]^N$ and whose range $[0,1]$ with the following properties: (1) For every $u \in [0,1]^N, C(u) = 0$ if 0 if at least one coordinate of u is 0 and if all coordinates of u are 1 except u_n, then. , $C(u) = u_n, n = 1,2, \ldots N$ (2) For every $a, b \in [0,1]^N$ such

that $a < b, V_c([a, b] \geq 0$

Sklar's theorem: Let H be a N −dimensional distribution function with 1 dimensional margins $F_1, F_2, \ldots . . F_N$. Then there exists a N −copulas C such that for all x in $\mathbb{R}^N$,

$$H(x_1, x_2, \ldots . , x_N) = C\big(F_1(x_1), F_2(x_2), \ldots . , F_N(x_N)\big). \qquad (4)$$

$$if\ F_1, F_2, \ldots , F_N\ are\ all\ continuous, the\ C\ is\ unique;$$

$$otherwise\ C\ is\ uniquely\ determined\ on\ RanF_1 \times RanF_2 \times \ldots . \times RanF_N.$$

Conversely, if C is a N −copula and $F_1, F_2, \ldots . . F_N$ are distribution functions, then the function H is defined by

$$H(x_1, x_2, \ldots . , x_N) = C(F_1(x_1), F_2(x_2), \ldots . , F_N(x_N); \theta)$$

is a $N - distribution\ function\ with\ margins$. Where θ is a parameter of the copula called the dependence parameter ,which measures dependence between the marginals.

The Sklar's theorem can be use to find copula when the margin and joint distribution are given.

Corollary. Let H be a N −dimensional distribution function with 1 dimensional margins $F_1, .. F_N$. Then there exists a N −copulas C such that for all x in $\mathbb{R}^N$ and $F_1^{(-1)}, \dots, F_N^{(-1)}$ be quasi-inverse of $F_1, .. F_N$, respectively. Then, for any $u\ in\ [0,1]^N$

$$C(u_1, u_2, \dots, u_N) = H(F_1^{(-1)}(u_1)F_2^{(-1)}(u_2),, \dots, F_N^{(-1)}(u_N))$$

If C is a copula and $F_1, .. F_N$ are univariate distribution functions, then Equation (4) above is a joint distribution function with margin $F_1, .. F_N$ (Tsay, 2013)

By applying Sklar theorem and exploiting the relation between the distribution and the density function, the multivariate copula density can easily be derive

$$c\big(F_1(x_1), F_2(x_2), \dots., F_N(x_N)\big)$$

associated with a copula function

$$C\big(F_1(x_1), F_2(x_2), \dots., F_N(x_N)\big):$$

The joint density function is obtained by differentiating once with respect to all arguments Equation(1) above and it is given the product of the marginals and copula density

$$h(x_1, x_2, \dots., x_N) = \frac{\partial^N[C\big(F_1(x_1), F_2(x_2), \dots., F_N(x_N)\big)]}{\partial F_1(x_1), \partial F_2(x_2), \dots., \partial F_N(x_N)} \prod_{n=1}^{N} F_n(x_n)$$

$$= c\big(F_1(x_1), F_2(x_2), \dots., F_N(x_N)\big) \prod_{n=1}^{N} F_n(x_n)$$

Defining

$$c\ \big(F_1(x_1), F_2(x_2), \dots., F_N(x_N)\big) = \frac{f(x_1, x_2, \dots., x_N)}{\prod_{n=1}^{N} F_n(x_n)}$$

Where $F_n$are the marginal densities that can different from each other (Tsay,2013;Cherubini et al 2004) which is related to the density function F for continuous random variables denoted by f by copula representation.

$$f(x_1, x_2, \dots., x_N) = c\big(F_1(x_1), F_2(x_2), \dots., F_N(x_N)\big) \prod_{n=1}^{N} F_n(x_n)$$

This allows to define the copula as a multivariate distribution with uniform $[0,1]$ margins

In financial applications, Cherubini et al 2011 discussed two most common used family of copulas: the elliptical and the Archimedean copulas. Elliptical copulas are symmetric which are derived from elliptical distribution by applying Sklar theorem. The commonly used are Gaussian and the Student's –t copulas. Their dependence structure is determined by a standard correlation or dispersion matrix because of the invariant property of copulas Archimedean copulas capture a wide range of dependence. Example of Archimedean copula are Clayton, Frank, and Gumbel. Frank copula is symmetric while Clayton and Gumbel copulas are asymmetric. Clayton captures lower tail dependence and Gumbel captures upper tail dependence. The general expression of Archimedean copulas for variables $u, v \epsilon [0,1]^2$ is

$$C(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v))$$

where φ is the generator which is strictly decreasing continuous function

φ^{-1} is the pseudo inverse.

3.3.1 Gaussian copula is defined as follows:

Let R be a symmetric, positive definite matrix with $diag(R) = 1$ and let $\Phi_R$the standardized multivariate normal distribution with correlation matrix R. Then the multivariate Gaussian copula is defined as

$$C^{Gauss}(u_1, u_2, \ldots u_N; R) = c(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \ldots \ldots \Phi^{-1}(u_N)$$

where Φ_R^{-1} is the inverse standard univariate normal distribution function Φ_R.

The associated multinomial copula density is

$$C^{Gauss}(\Phi(x_1), \Phi(x_2), \ldots \ldots, \Phi(x_N); R) = \frac{f^{Gauss}(x_1, x_2, \ldots., x_N)}{\prod_{n=1}^{N} f_n^{Gauss}(x_n)}$$

$$= \frac{\frac{1}{2\pi^{\frac{N}{2}}|R|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}x' R^{-1} x\right)}{\prod_n^N \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x_n^2\right)},$$

fixing $u_n = \Phi(x_n)$, and denote

$$\varsigma = (\Phi^{-1}(u_1), \Phi^{-1}(u_2), \ldots., \Phi^{-1}(u_N)')$$

the vector of the Gaussian univariate distribution function is

$$c(u_1, u_2, \ldots u_N; R) = \frac{1}{|R|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}\varsigma'(R^{-1} - 1)\varsigma\right].$$

3.3.2 The BB7 copula is defined as

Let K be the bivariate Gumbel copula and let ψ be the gamma $LT: \psi_\theta = (1+S)^{-1/\theta}$ then

$$C(u_1, u_2) = (1 + [(u_1^{-\theta} - 1)^{\delta} + u_2^{-\theta-1})]^{\frac{-1}{\theta}})$$

With lower tail dependence (λ_L) = $2^{-1/(\delta\theta)}$, uper tail dependence (λ_U) =$2 - 2^{1/\delta}$

3.3.3 The student t copula is defined as

Let R be a symmetric, positive definite matrix with $diag(R) = 1$ and let $T_{R,v}$ the standardized multivariate student t distribution and the correlation matrix R and v degree of freedom. Then the multivarite1 student t copula is defined as follows

$$c(u_1, u_2, \ldots u_N; R, v) = T_{R,v}(t_v^{-1}(u_1), (t_v^{-1}(u_2), \ldots., (t_v^{-1}(u_N)),$$

where $t_v^{-1}(u_n)$ is the inverse of the student t cumulative distribution function.

The associate Student t copula density is

$$c(u_1, u_2, \dots u_N; R, v) = \frac{f^{student}(x_1, x_2, \dots, x_N)}{\prod_{n=1}^{N} f_n^{student} x_n}$$

$$= |R|^{-\frac{1}{2}} \frac{\Gamma(\frac{v+N}{2})}{\Gamma(\frac{v}{2})} [\frac{\Gamma(\frac{v}{2})}{\Gamma(\frac{v+1}{2})}]^N \frac{(1+\frac{(\varsigma' R^{-1} \varsigma)}{v})^{r-\frac{v+N}{2}}}{\prod_{n=1}^{N}(1+\frac{\varsigma_n^2}{v})^{-\frac{v+1}{2}}},$$

where $\varsigma = (t_v^{-1}(u_1), t_v^{-1}(u_2), \dots, t_v^{-1}(u_N))'$.

3.4 Estimation of Value at Risk Using Copula

Copula to estimate VaR of a portfolio consists of several assets including $AR(1) - GARCH(1,1) +$ Gaussian copula and $AR(1) - GARCH(1,1) +$ student t copula. In these models, each return series is assumed to follow $AR(1) - GARCH(1,1)$ models and innovations are simultaneously generated using copulas. It is also involving estimation of multivariate model. There are two steps of estimating multivariate models using copula.

(1) There is a two-step procedure for the identification and estimation of the joint CDF

(a) identification and estimation of the marginals.

(b) identification and estimation of the copula function.

This is also referred to as inference for margins (Dias,2004). The first step of the IFM (inference for margin) method requires choosing a family of distributions to model each univariate return time series, independently of any copula mode.The marginal distributions and the copula need not belong to the same family of distributions because Sklar's theorem has enabled a lot of flexibility in multivariate modelling (2) one-stage maximum likelihood estimates are obtained by maximing sum of copula likelihood functions for all observation in one step by plugging all the required parameters into the copula.

3.4.1 Modelling the marginal distributions

ARMA-GARCH models has been successfully modelled returns series. AR(1)-GARCH(1, 1) model are used to model the margins as follows

$$x_{n,t} = \mu_n + \emptyset_n x_{n,t-1} + \epsilon_{n,t};$$
$$\epsilon_{n,t} = \sigma_{n,t} \eta_{n,t};$$
$$\sigma_{n,t}^2 = \alpha_n + \beta_n \epsilon_{n,t-1}^2 + \gamma_n \sigma_{n,t-1}^2;$$

where $\{ \eta_{n,t} \}$ is white noise process, α_n , β_n, γ_n satisfy the condition of GARCH model: $\beta_n + \gamma_n < 1$ for n = 1, 2,...,N and $t = 1, 2, \dots, T$. The conditional distribution of the standardized innovations

$$\eta_{n,t} = \frac{\epsilon_{n,t}}{\sigma_{n,t}} | \Im_{n,T}, \quad n = 1,2, \dots, N,$$

was modelled by white noises and denoted by $F_{n,t}$ in general case (the marginal distributions). Considering the case that $\eta_{n,t}$ are standard normal distributions and student *t* distributions with the same degree of freedom, $n = 1,2, \dots, N$.

The joint distribution of innovation vector $\eta_t = \eta_{1,t}, \eta_{2,t}, \dots, \eta_{N,t}$ is model by copula.

Let $\eta_{n,t} = F_{n,t}(\eta_{n,t}|\Im), F_{1,t}, F_{2,t}, \ldots . and\ F_{N,t}$ are marginal distributions conditioned to $\Im$, the information available up to time T-1. If the models were correctly specified then series $\{u_{n,t}|t = 1,2, \ldots, T-1\}$ will be standard uniform series.

3.4.2 Modelling the copula

Assuming that $(\eta_{1,T}, \eta_{2,T}, \ldots, \eta_{N,T})$ has multivariate distribution function

$$H_T(\eta_{1,T}, \eta_{2,T}, \ldots, \eta_{N,T}; \theta_{1,T}, \theta_{2,T}|\Im)$$

are continuous univariate marginal distribution functions $F_{n,T}(\eta_{n,T}; \theta_{n,T}|\Im)$

where $\Im = \eta_{n,t}|n = 1,2, \ldots . N, t = 1,2, \ldots, T-1\}$.

Since the marginal distributions are continuous, the conditional copula C_T is uniquely defined according to Sklar theory, having

$$C_T\,(F_{1,T}(\eta_{1,T}; \theta_{1,T}|\Im), F_{2,T}(\eta_{2,T}; \theta_{2,T}|\Im, \ldots, F_{N,T}(\eta_{N,T}; \theta_{N,T}|\Im); \theta_{2,T}|\Im)$$
$$= H_T(\eta_{1,T}, \eta_{2,T}, \ldots, \eta_{N,T}; \theta_{1,T}, \theta_{2,T}|\Im)$$

Where $\theta_{1,T}$ is the margins' parameters and $\theta_{2,T}$ is the copula's function C_T

The parameters $\theta_{1,T}$, $\theta_{2,T}$ are estimated by using IFM (inference for the margins) method as follows

1. Estimation of the margin's parameters $\widehat{\theta_{1,T}}$ by performing the estimation of the univariate marginal distribution.

$$\widehat{\theta_{1,T}} = argmax \sum_{t=1}^{T-1} \sum_{n=1}^{N} lnf_{n,T}(\eta_{n,t}; \theta_{1,T}).$$

2. Estimation of the copula parameters $\widehat{\theta_{2,T}}$ given $\widehat{\theta_{1,T}}$

$$\widehat{\theta_{2,T}} = argmax \sum_{t=1}^{T-1} lnC_T\,(F_{1,T}(\eta_{1,t}; \theta_{1,T}), F_{2,T}(\eta_{2,t}; \theta_{1,T}), \ldots, F_{n,T}(\eta_{n,t}; \theta_{1,T}); \theta_{2,T}).$$

If the marginal distribution $F_{n,T}$ are standard normal distributions, then C_T is a multivariate Gaussian copula with correlation matrix $\theta_{2,T} = R_T$. In this case, N marginal distributions are assumed to have the same degree of freedom.

3.5 COPULA-BASED DEPENDENCE MEASURE

Copula-based measure of dependence, as defined by McNeil, et al. (2005), is coefficient of tail dependence which measures the strength of dependence in the tails.

One of the most commonly used coefficient of rank correlation is Kendall's τ. It relies on the notion of concordance. A pair of random variables is *concordant* whenever large values of one variable are associated with large values of the other variable. More formally, if (y_i, x_i) and (y_j, x_j) are two observations of random variables (Y, X). Then, the pairs are *concordant* whenever $(y_i - y_j)(x_i - x_j) >$ and *discordant* whenever $(y_i - y_j)(x_i - x_j) < 0$.

Kendall's τ is defined as the difference between the probability of concordance and the probability of discordance. Kendall's τ is a copula-based dependence measure in the sense that it does not depend on the marginal distribution, but is exclusively a function of the copula:(note i.e. the copula can be used to measure the τ) based on (Nelsen, 2006)

$$\tau = \frac{c-d}{c+d}$$

$$\tau_{u_1,u_2} = 4 \int \int_{[0,1]^2} C(u_1, u_2) dC(u_1, u_2) - 1$$

where $c(d)$ are the number of concordant (discordant) pairs, C is the copula, u_1 and u_2 are the values of the CDFs. The second advantage of using a copula is that one can measure tail dependence , which measures the probability that two variables are in the lower or upper joint tails. The coefficient of tail dependencies, in this case, a measure of the tendency of markets to crash or boom together. The coefficients of lower and upper tail dependence (λ_L and λ_U) can be expressed in terms of the copula between X and Y.Based on (Nelsen,2006).
Kendall's τ measures overall dependence, there exist copula-based measures of dependence that focus on dependence between extremes.
Quantile dependence focuses on the tails of the distribution. If X and Y are random variables with distribution functions FX and FY , there is quantile dependence in the lower tail at threshold u, whenever $P[Y \leq F-1Y (u)|X \leq F-1X (u)]$ is different from zero.
Finally, **tail dependence** is obtained as the limit of the probability.
The coefficient of lower tail dependence of X and Y is

$$\lambda_L = \lim_{u\to 0^+} P[Y \leq F_Y^{-1}(u)|X \leq F_X^{-1}(u)] = \lim_{u\to 0^+} \frac{C(u,u)}{u}$$

$$\lambda_U = \lim_{u\to 1^-} P[Y \leq F_Y^{-1}(u)|X \leq F_X^{-1}(u)] = \lim_{u\to 1^-} \frac{1-2u+C(u,u)}{1-u}$$

where $F_Y^{-1}$and $F_X^{-1}$are the marginal quantile functions and where λ_L and $\lambda_U \epsilon [0,1]$Roughly speaking, $\lambda_L(\lambda_U)$measures the probability that $Y$is below (above) a low (high) quantile, given that X is below (above) a low (high) quantile. If λ_L $or \lambda_U$ is positive, then there is lower or upper tail dependence, otherwise there is lower or upper tail independence. Again, there is a symmetric tail dependence between two assets when $\lambda_L = \lambda_U$, otherwise it is asymmetric. Different copulas usually represent different dependence structures with the association parameters indicating the strength of dependence.

3.5.1 The Gumbel copula (1960) is used to model asymmetric dependence in the data. This copula is famous for its ability to capture strong upper tail dependence and the weak lower tail dependence. If outcomes are expected to be strongly correlated at high values but less correlated at low values then the Gumbel copula is an appropriate choice. The bivariate Gumbel is given by:

$$C^{Gu}(u_1, u_2, \theta) = \exp\,[-((-logu_1)^\theta + (-logu_2)^\theta)^{\frac{1}{\theta}}]$$

Where θ is the copula parameter restricted on the interval [1,∞]. When θ approaches 1, the marginal become independent and when θ goes to infinity the Gunbel copula approaches the Fretchet-Hoeffding upper bound. Like the Clayton copula, the Gumbel copula represents only the case of independence and positive dependence.
The relation between the Gumbel copula parameters and the Kendall's tau is given by:

$$\tau_k = 1 - \theta^{-1}$$

The parameter of the upper and lower tail dependence of the Gumbel copula can be calculated respectively by $\lambda_u = 2 - 2^{\frac{1}{\theta}}$, which is the upper tail dependency, and $\lambda_L = 0$, It has no lower tail dependency. If $\theta = 1$, implies the independent copula and if $\theta \to \infty$, implies the minimum copula. Gumbel copula can be applied when dependency increases with extreme positive values especially during credit portfolio risk and when dependency increases during times of recession and credit losses.

3.5.2 Clayton Copula

The Clayton Copula was first introduced by Clayton (1978). The Clayton copula is mostly used to study correlated risks because of their ability to capture lower fail dependence. The closed form of the bivariate Clayton copula is given by:

$$C^{cl}(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}}$$

Where θ is the copula parameter restricted on the interval $(0, \infty)$. If θ=0 then marginal distributions become independent and when θ goes to infinity the Clayton copula approximate the Fretchet-Hoeffding upper bound .Due to the restriction on the dependence parameter, the Fretchet-Hoeffding upper bound cannot be reached by the Clayton copula. This suggests that the Clayton copula cannot account for negative dependence. The dependence between the Clayton copula parameter and Kendall's tau rank measure is simply given by

$$\tau_k = \frac{\theta}{\theta+2}$$

The parameter of lower tail dependence for this copula can be calculated by: $\lambda_L = 2^{-\frac{1}{\theta}}$ (Cherubini, Lucino and Vecchiatar, 2004). Clayton copula has lower tail dependency but has no upper tail dependency.

3.5.3 Frank Copula

Frank Copula (1979) is given by $c^{Fr}\left(u_{1,} u_{2,} \theta\right) = \theta^{-1}\log\,[1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{(e^{-\theta} - 1)}]$

Where θ is the copula parameter that may take any real value i.e $\theta \in \mathbb{R}\backslash\{0\}$. Unlike the Clayton and the Gumbel copula, the Frank copula allows the maximum range of dependence. This means that the dependence parameter of the Frank copula permits the approximation of the upper and the lower Fretchet-Hoeffding bounds and thus the Frank copula permits modeling positive as negative dependence in the data. The relation between the Frank copula parameters and the Kendall's tau is given by.

$$\tau_k = 1 - \left(\frac{4}{\theta}\right) + 4\left(\frac{D_1(\theta)}{\theta}\right)$$

The dependence from Frank's copula relies on the $Debye\ functions$ defined as

$$D_1(\theta) = \frac{1}{\theta}\int_o^\theta \frac{t}{\exp(t) - 1}dt \quad for\ \theta > 0.$$

To evaluate the negative arguments of the $Debye\ function\ D_1$, the basic calculus shows that.

$$D_1(-\theta) = D_1(\theta) + \frac{\theta}{2}$$

When θ approaches +∞ and -∞ the Fretchet-Hoeffding upper and lower bound will be attained. The independence case will be attained when θ approaches zero. However, the Frank copula has neither lower nor upper tail dependence ($\lambda_U = \lambda_L = 0$). The Frank copula is thus suitable for modelling data characterized by weak tail dependence.

3.6 MIXTURE COPULA

Gaussian copula has zero tail dependence while Clayton copula has left tail dependence and no right tail dependence. On the contrary, Gumbel copula has right tail dependence and no left tail

dependence. Each of the copulas has both advantages and disadvantages. The Archimedean copulas are restricted to show either dependence of negative or positive joint.
events. It may be that the more dimensions the data set has, the more unlikely it is that all variables share the same dependence: a prerequisite for a good fit of copulas with just one dependence parameter. The elliptical copula model meanwhile model's dependence through
a greater number of parameters but is limited to symmetric dependence. To combine the advantageous features of both a new class of copulas has been introduced. These are known as'mixture copulas'. The mixture allows the use of two or more copulas to describe the dependence structure of a data set. Clearly, the mixing of different copulas can generate a wide range of dependence structures. A bivariate mixture copula may be written as$C^{mix}(u_1, u_2, \ldots., u_N; \theta_1, \theta_2, m_1, m_2)$

$$= m_1 . C^{mix1}(u_1, u_2, \ldots., u_N; \theta_1) \; + m_2 . C^{mix2}(u_1, u_2, \ldots., u_N; \theta_2)$$

where $m_2 = 1 - m_1, C^{mix}$ represent the mixture copula, C^{mix1} is the first copula in the mixture copula and C^{mix2} is the second copula in the mixture copula. The two-step MLE procedure can be use estimate the mixture copula which is also known as the method of Inference Function for Margins(IFM).

3.7 Estimation and Inference

Sklar's theorem opens the way for two alternative estimation methods for copulas based on the likelihood. The estimation of the ARMA-GARCH process and the copula are separated regardless of the copula model which has resulted in two great advantages. Firstly, the marginal models are estimated once regardless of the copula function, this has reduce the number of parameters to be estimated in each step. Secondly, the residuals that set for basis for any copula estimation are filtered by the same marginal models. All copulas are calibrated on the same data, which allows a direct comparison of the copulas and the copulas are not biased based by differences in the marginals. The concept of conditional copula was introduced by Patton (2006b) which allowing for time-variation in the parameters of the marginal distributions. It is particularly useful for returns, since volatility models imply that marginal distributions have time-varying means and volatilities. The conditional copula is

$$F_t = (y_{1t}, \ldots., y_{nt} | \boldsymbol{Y}^{t-1}) = C_t(F_{1t}(y_{1t} | \boldsymbol{Y}^{t-1}), \ldots., F_{nt}(y_{nt} | \boldsymbol{Y}^{t-1}) | \boldsymbol{Y}^{t-1})$$

where $Y_s = (y_{1s}, \ldots, y_{ns})$ denotes the time s observations of all series and $\boldsymbol{Y}^{t-1} = [Y_s]_{s=1}^{t-1}$ is the history of the multivariate process up to time $t-1$. The density is obtained by differentiating and taking logs of the conditional copula leading to the joint likelihood of the marginal and the copula, the total log-likelihood (LL) depends on all the data ($\boldsymbol{Y}^{t \times d}$) and is written as

$$LL(\boldsymbol{Y}: \theta_m, \theta_c) = \sum_{t=1}^{T} \log f(\boldsymbol{Y}_t | \boldsymbol{Y}^{t-1}; \theta_m, \theta_c)$$

Where θ_m denotes the parameters of the marginals, θ_c the copula parameters and
$\boldsymbol{Y}^{t-1} = (\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_t)$ represents the history of the full process. Consequently, l can decompose the log- likelihood into one part for the marginals (LL_m) and one part for the copula (LL_c) :

$$LL(\boldsymbol{Y}; \theta_m, \theta_c) = LL_m(\boldsymbol{Y}; \theta_m) + LL_c(\boldsymbol{Y}; \theta_m, \theta_c)$$

$$LL_m(\boldsymbol{Y}; \theta_m) = \sum_{t=1}^{T} \sum_{i=1}^{n} log f_i(x_{i,t} | x_i^{t-1}; \theta_{m,i})$$

$$LL_c(\boldsymbol{Y};\theta_m,\theta_c)=\sum_{t=1}^{T} log\ c(F_1(y_{1,t}|y_1^{t-1};\theta_{m,1}),\dots,F_n(y_{n,t}|x_n^{t-1};\theta_{m,n});\theta_c)$$

if each variable i only depend on its own history, $x_i^{t-1}=(x_{i,1},\dots,x_{i,t})$. The likelihood of the marginal models (LL_m) is a function of the parameter vector $\theta_m=(\theta_{m,1},\dots,\theta_{m,n})$, that collects the parameters of each one of the n marginal densities f_i, but the copula likelihood directly depends on the copula parameter θ_c and indirectly on the parameters of the marginal densities, through the distribution function F_i because F_i transforms the observations into uniform [0,1] variables that are the inputs for the copula. The two step estimation procedure is also known as the method of inference Functions for Margins (IFM) as discussed in (Joe&Xu,1996). Fortunately, Newey and McFadden (1994) showed that one and two-step estimations are similarly efficient. Assume that the marginals depend only on their own history, but are independent from each other. Therefore, the margins can be estimated:

$$\hat{\theta}_m=argmax_{\theta_m}\sum_{t=1}^{T}\sum_{n=1}^{n}\log\ f_i(y_{i,t}|y_i^{t-1};\theta_{m,i})$$

Because the marginal models are independent of each other (i.e. when the univariate ARMA-GARCH model are used), it is simplified further into a set of separate estimations for each one of the magin i, so that estimate of each series can be separated as in

$$\hat{\theta}_{m,i}=argmax_{\theta_{m,i}}\sum_{t=1}^{T} logf_i(y_{i,t}|y_i^{t-1},\theta_{m,1})$$

and collect the coefficients in a vector: $\hat{\theta}_m=(\hat{\theta}_{m,1},\dots.,\hat{\theta}_{m,n})$ and in a second step, the parameters of the copula are estimated taking as given the parameter estimates of the marginal models:

$$\hat{\theta}_c=argmax_\theta\ LL_c(Y;\hat{\theta}_m,\theta_c)$$

To calculate the copula standard errors, the IFM estimator ($\hat{\theta}_{IFM}$) collects all the parameters of the marginal and copula methods as a vector, $\hat{\theta}_{IFM}=(\hat{\theta}_{m,1},\dots,\hat{\theta}_{m,n},\hat{\theta}_c)$. Durrleman et al. (2000) propose an approach to calculate the copula standard errors based on the Godambe information ratio. They show that the IFM estimator verifies the property of asymptotics normality as shown below

$$\sqrt{T}\left(\hat{\theta}_{IFM}-\theta_0\right)\rightarrow N\left(0,G^{-1}(\theta_0)\right)$$

With $G(\theta_0)$ the information matrix of Godambe. They further define a score function, $sc(\theta)=\left(\partial\theta_{m,1}LL_{M,1},\dots,\partial\theta_{m,n}LL_{m,d},\partial\theta_c LL_c\right)$. The Godambe information matrix takes the form (Joe 1997),

$$G(\theta_0)=D^{-1}V(D^{-1})'$$

where $D=E[\frac{\partial sc(\theta)'}{\partial\theta}]$ and $V=E[sc(\theta)'sc(\theta)]$. Joe and Xu (1996) suggest the jackknife resampling method (the most commonly used method) for estimation of the variance and efficient estimation, because the estimation of the covariance matrix requires one to calculate many derivatives.

3.8 Value at risk

The maximum likelihood estimates MLEs ($\hat{\beta}$,$\hat{\sigma}$, $\hat{\varepsilon}$) for a GPD, threshold u and N_u the number of exceedances is used to quantify the value at risk, for tail probability P, and total sample size n. VaR is given by

$$V\hat{a}R_p(y_t)=\begin{cases} u+\frac{\hat{\beta}}{\hat{\varepsilon}}[1-\{-nlim(1-p)^{-\hat{\varepsilon}}\}] if \hat{\varepsilon}\neq 0 \\ u-\hat{\sigma}\ \ln(-nln(1-p)) \quad if\ \hat{\varepsilon}=O \end{cases}$$

for a GEVD with maximum likelihood estimates $(\hat{\mu},\hat{\sigma},\hat{\varepsilon})$,

$$V\hat{a}R_p(y_t)=\begin{cases} u+\frac{\hat{\sigma}}{\hat{\varepsilon}}\{\left(\frac{n}{N_u}p\right)^{-\hat{\varepsilon}}-1\} \ if\ \hat{\varepsilon}\neq 0 \\ u+\hat{\beta}\ln\left(\frac{n}{N_u}(1-p)\right. \quad if\ \hat{\varepsilon}=o \end{cases}$$

The VaR of the asset is computed finally using the following formula:

$$VaR_p(r_t)=\sigma_{kt}+\sigma_{kt}.VaR_p(y_t)$$

where $VaR_p(r_t)$ is the p percentile of the standardised residuals. The return σ_{kt} is estimated from the volatility model. The riskiness of the asset is expressed through $VaR_p(y_t)$

3.9 Monte Carlon simulation

Using Gaussian and student t copula to simulate k vector

$$\eta_{T,k}=(\eta_{1,T,k},\eta_{2,Tk},\dots,\eta_{N,T.k}),$$

for $k=1,2,\dots,K$

The Monte Carlon simulation process for multivariate Gaussian copula is as follow:

1. Find the Cholesky decomposition A of the linear correlation matrix R.
2. Simulate N i.i.d. $z=(z_1,z_2,\dots,z_N)'$ from $N(0,1)$
3. Simulate a random variate s from χ_v^2 independent of z
4. Set $\eta'_{T,k}=Az$

The Monte Carlon simulation for multivariate student t copula

1. Find the Cholesky decomposition A of the linear correlation matrix R
2. Simulate N i.i.d. $z=(z_1,z_2,\dots,z_N)'$ from $N(0,1)$
3. Simulate a random variate s from χ_v^2 independent of z
4. Set $y=Az$
5. Set $\eta'_{T,k}=\sqrt{\left(\frac{v}{s}\right)}y$

Simulating K vectors $(x_{1,T,k},x_{2,T,k},\dots,x_{N,T,K})$ and K values of $x_{T,k}$ by using the model below: $x_{n,t}=\mu_n+\emptyset_n x_{n,t-1}+\epsilon_{n,t}$;

$\epsilon_{n,t}=\sigma_{n,t}\eta_{n,t}$;

$\sigma_{n,t}^2=\alpha_n+\beta_n\epsilon_{n,t-1}^2+\gamma_n\sigma_{n,t-1}^2$,

for $k=1,2,\dots,K$. Ordering series $\{x_{T,k}\}$ in increasing order. The VaR of portfolio by $VaR_T(\alpha)=x_{T,K_p}$.

4.0 RESULT AND DISCUSSION

Table 4.1: Comparison of copula models

Pairs	Parameter	Gaussian	Frank	Gumbel	Clayton	BB7	t copula
Dan-GTCO	θ	0.28	1.53	1.2	0.35		
	L-L	21.98	16.37	24.48	20.63	**28.39**	27.26
	AIC	-41.87	-30.73	-46.97	-39.26	**-52.79**	-50.51
	BIC	-37.55	-26.42	-42.66	-34.95	**-44.16**	-41.89

Dan-Nestle	θ	0.26	1.48	1.2	0.32		
	L-L	18.91	14.29	22.92	16.79	**29.81**	29.81
	AIC	-35.81	-26.59	-43.85	-3.59	**-55.62**	-55.62
	BIC	-31.5	-22.27	-39.53	-27.28	**-47.00**	-47.00
Dan-Vit	θ	0.15	0.99	1.12	0.23		
	L-L	6.13	6.84	8.9	8.87	**11.3**	8.82
	AIC	-10.26	-11.69	-15.79	-15.75	**-18.6**	-13.63
	BIC	-5.95	-7.38	**-11.48**	-11.44	-9.97	-5.01
Dan-Fid	θ	0.2	1.09	1.13	0.23		
	L-L	11.2	8.52	11.11	9.22	**12.88**	11.73
	AIC	**-20.24**	-15.05	-20.22	-16.44	-12.88	11.73
	BIC	**-15.93**	-10.74	-15.91	-12.13	-13.14	10.83
GTCO-Nes	θ	0.23	1.21	1.14	0.22		
	L-L	13.72	10.06	13.56	9.17	**19.94**	18.2
	AIC	-25.43	-18.12	-25.11	-16.35	**-35.87**	32.4
	BIC	-21.12	-13.81	-20.8	-12.04	**-27.25**	23.78
GTCO-VIT	θ	0.09	0.6	1.05	0.1	0.6	
	L-L	2.14	**2.73**	1.62	2.09	2.70	2.41
	AIC	-2.28	-3.46	-1.23	-2.18	**-3.46**	0.81
	BIC	2.03	0.85	3.08	2.13	**0.85**	7.81
GTCO-FID	θ	0.15	0.76	1.09	0.17	**1.09**	
	L-L	6.02	4.32	6.23	5.59	**6.23**	
	AIC	-10.05	-6.65	-10.45	-9.18	**-10.45**	5.71
	BIC	-5.73	-2.34	-6.14	-4.87	**-6.14**	1.2
NEST-VIT	θ	0.1	0.64	1.08	0.13	1.08	
	L-L	2.67	2.77	4.13	3.3	**4.13**	5.11
	AIC	-3.33	-3.54	-6.25	-4.59	**-6.25**	6.21
	BIC	0.98	0.77	-1.94	-0.28	**-1.94**	2.41
NEST-FID	θ	0.09	0.73	1.09	0.17	1.09	
	L-L	5.47	3.71	6.56	6.29	**6.56**	7.04
	AIC	-8.93	-5.43	-11.12	-10.57	**-11.12**	10.09
	BIC	-4.62	-1.12	**-6.81**	-6.26	-6.56	1.47
VITA-FID	θ	0.2	1.14	1.13	0.22		
	L-L	10.61	8.98	10.21	8.44	**12.18**	12.18
	AIC	-19.22	-15.97	-18.43	-14.87	**-20.36**	20.36
	BIC	**-14.91**	-11.65	-14.12	-10.56	-11.74	-11.74

Note: The bold values in the table refer to the LL,AIC and BIC for the best fitted copula model. The AIC criterion and binary segmentation procedure (BIC) are used to find the copula that best fits the data set and determine the change points of the copula family and copula parameters.

Table 4.2: Tail Dependence Coefficient of Best Copula BB7 Model

Pair	Lower tail dependence (λ_L)	Upper tail dependence (λ_U)	Kendall's tau
DAN-Gtco	0.06	0.20	0.18
DAN-Nestle	0.15	0.15	0.16
DAN-Vitaform	0.00	0.00	0.12

DAN-Fidson	0.02	0.13	0.13
GTCO-Nestle	0.11	0.06	0.15
GTCO-Vitaform	0.00	0.00	0.07
GTCO-Fidson	0.11	0.00	0.09
NEST-Vitaform	0.10	0.00	0.07
NEST-Fdson	0.00	0.11	0.08
VITA-FID	0.02	0.02	0.12

The estimated values are reported in table 2. In the financial data context upper tail dependence means dependence in boom market, while lower tail dependence means dependence in bear market. It is observed that dependences in boom market are different than the dependences in bear market. Since upper tail dependence parameters are higher than lower tail dependence parameter for some pairs like DAN-Gtco ,DAN-Fidson ,NEST-FID, which are more likely to rise together than to fall together. It is noted the pair is the strongest tail dependent pair for both positive and negative co-exceedances. The NEST-Vitaform and GTCO-Fidson, pairs show the smallest degree of upper tail dependence, while DAN-Vitaform, GTCO-Vitaform and NEST-Fidson pairs report the smallest degree of lower tail dependence. To interpret the tail dependence results, taking the upper tail and the lower tail dependence parametersλ_U and λ_L for DAN-Gtco pair. For this pair, λ_U is estimated to be 0.20 meaning that given DAN having a price jump above a certain value, the probability of Naira having a price jump above a corresponding value is about 20%.The lower tail dependence parameterλ_L for the pair is estimated to be 0.06, meaning that given DAN having a price drop below a corresponding value is about 6%. The dependence structure exhibits the asymmetric dependence between two variables. The degree of asymmetry in the dependence in the upper and lower tail reported in Table 2 shows that two variable exhibit greater correlation during market upturns than market downturn.

Table 4.3: Parameter estimates for GPD model.

Statistics	μ	φ	ξ
DANCEM	0.4326	0.1513	-0.2035
GTCO	0.5573	0.2539	-0.2258
NESTLE	0.5570	0.2398	-0.5764
VITAFOAM	1.4884	0.9891	-1.008
FIDSON	10082	0.5670	-0.2842

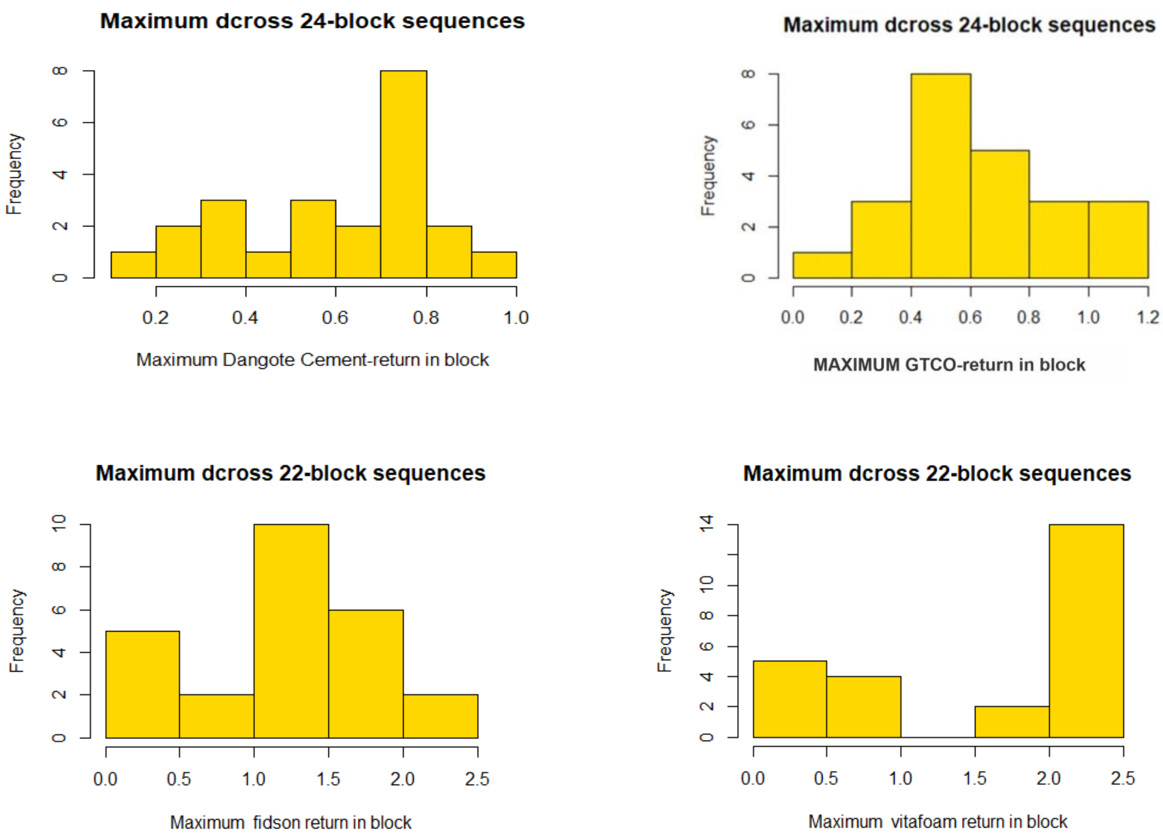
The standardised residuals of the selected best fit model were extracted and used to fit Generalised Pareto Distributions and Generalised Extreme Value Distributions before being used in the estimation of the VaR. From Table 3, it is observed that the shape parameter ξ values for all the variables are significantly negative which have short tailedness for left tail. It is also observed that vitafoam has insignificant short tailedness. Figure 2 shows the qq plots of the lower and upper tail exceedances against the quantiles obtained from the GPD fit. The approximate linearity of the plots indicates the GPD model seems tobe a good choice.

Table 4.4: Value-at-risk estimates

Statistics	Confidence Interval	Copula family	
		Archimedean	Elliptical
		Gumbel	Student t
DANCEM	95%	0.970850	0.9741139
GTCO	95%	1.2053858	1.1965139
NESTLE	95%	0.9864471	0.9887426
VITAFOAM	95%	1.2356083	1.2382889
FIDSON	95%	0.6711793	0.4552257

VaR estimate based on the selected ARCHIMEDIAN and ELLIPTICAL copulas for the constructed Portfolio. The VaR estimates presented in Table 4 clearly show that during global financial crisis, the risk of collapse in some selected stock in Nigeria was extremely high.

Figure 1



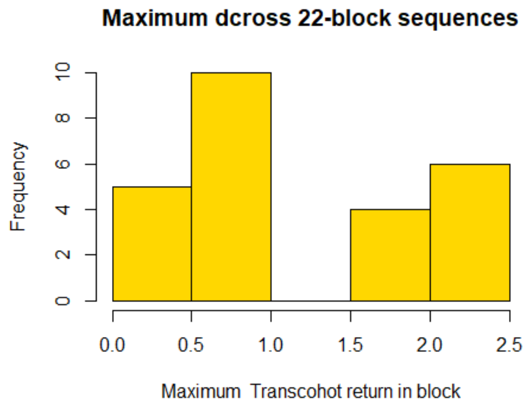
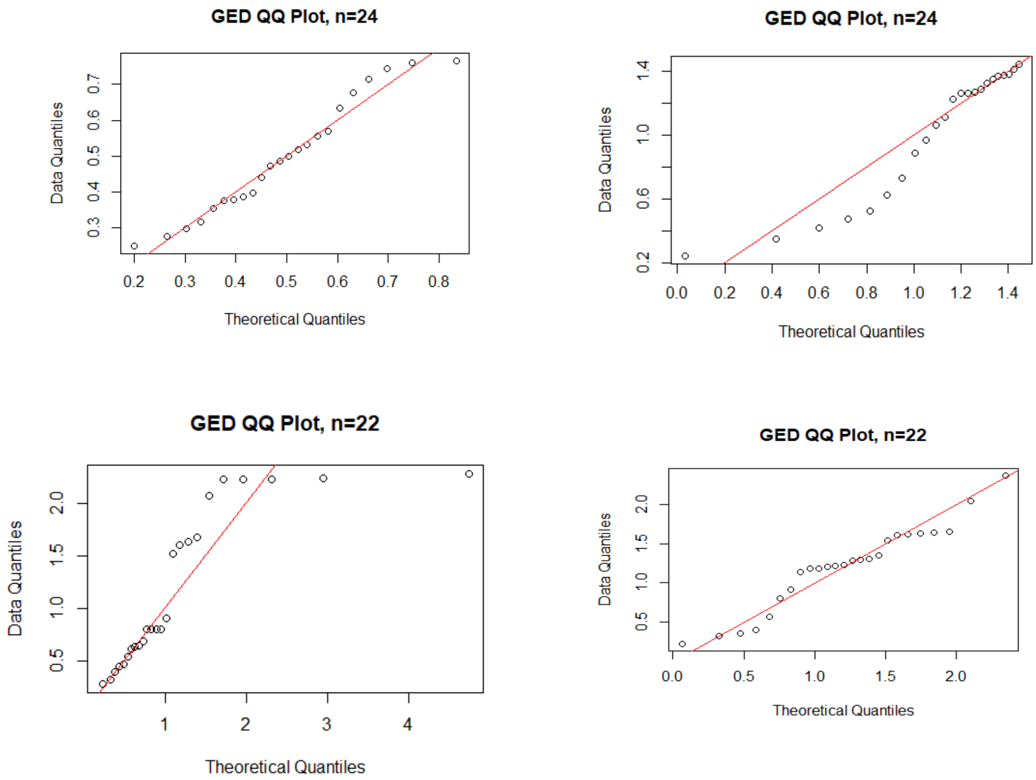

Maximum dcross 22-block sequences
Frequency
0
2
4
6
8
10
0.0
0.5
1.0
1.5
2.0
2.5
Maximum Transcohot return in block

Figure 2


GED QQ Plot, n=24
Data Quantiles
Theoretical Quantiles
GED QQ Plot, n=24
Data Quantiles
Theoretical Quantiles
GED QQ Plot, n=22
Data Quantiles
Theoretical Quantiles
GED QQ Plot, n=22
Data Quantiles
Theoretical Quantiles

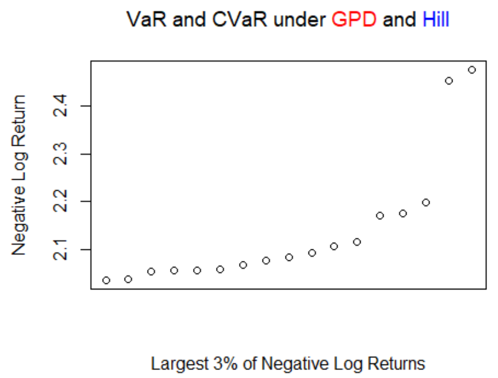
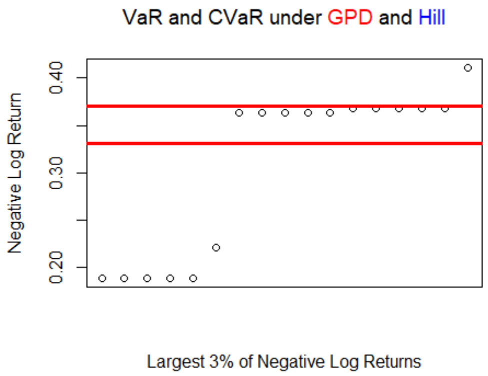
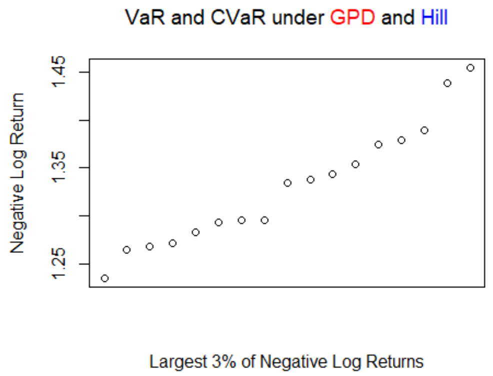
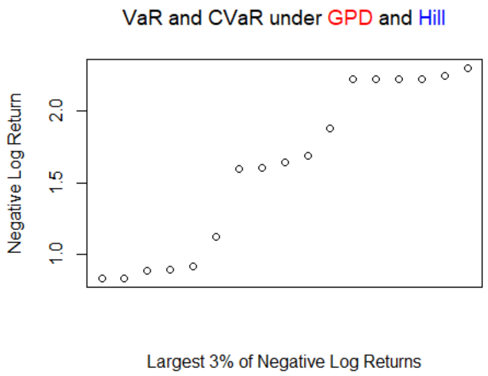
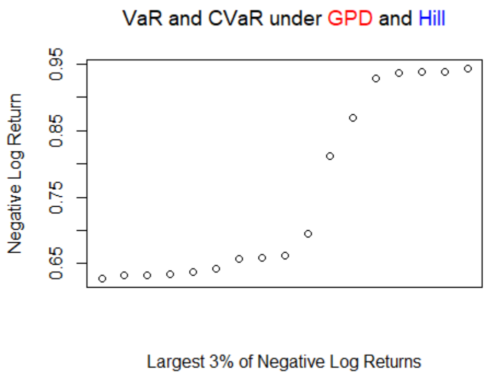

VaR and CVaR under GPD and Hill
Negative Log Return
2.1
2.2
2.3
2.4
Largest 3% of Negative Log Returns

Figure 3


VaR and CVaR under GPD and Hill
Negative Log Return
0.20
0.30
0.40
Largest 3% of Negative Log Returns


VaR and CVaR under GPD and Hill
Negative Log Return
1.25
1.35
1.45
Largest 3% of Negative Log Returns


VaR and CVaR under GPD and Hill
Negative Log Return
1.0
1.5
2.0
Largest 3% of Negative Log Returns


VaR and CVaR under GPD and Hill
Negative Log Return
0.65
0.75
0.85
0.95
Largest 3% of Negative Log Returns

Conclusion

In this paper, we examined dependence structure and Portfolio Value-At-Risk for some selected Nigerian stocks using a Copula-Based Volatility Model. We performed a range of goodness-of-fit tests to help us select the best copula as a measure of dependence between Nigerian stock markets. Marginal model for the stock returns and a joint model for the dependence for the dependence were specified. EVT model was employed for the marginal distribution of each return series, and for the joint model, the family of copula such as Gaussian, Frank, Gumbel, Clayton, BB7, Student-t copula was used with difference dependence structure, Using LL, AIC, and BIC values, BB7 was found to be the best fitted copula. Copula was used to measure Portfolio risk and global minimum risk portfolio is selected based on efficient frontiers. In estimating VaR, precise specification and identification of the probability of an extreme movement in the value of an individual asset (or portfolio) is essential for risk assessment. The evidence has direct implications for investors and risk managers during extreme currency market movements.

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